

# chapter 3 single degree of freedom systems springer

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## Introduction

When studying mechanical vibrations, one of the most foundational concepts is the single degree of freedom (SDOF) system. These systems capture the essential dynamics of many engineering structures while remaining mathematically tractable. Whether you are a student tackling your first vibration assignment, a researcher validating a new damping material, or an engineer designing a precision instrument, understanding SDOF dynamics is indispensable.

This article delves deeply into the theory and application of SDOF systems, guided by the structure and insights found in **Chapter 3: Single Degree of Freedom Systems** of a prominent Springer textbook on vibrations. We will explore the mathematical modeling, solution techniques, and practical examples that make this chapter a cornerstone for anyone working in dynamic analysis.

## What Is a Single Degree of Freedom System?

A single degree of freedom system is a mechanical model that can be described by one independent coordinate. In the context of vibrations, the coordinate typically represents the displacement of a mass relative to a fixed reference. Classic examples include:

- The mass–spring–damper system.
- The vertical motion of a vehicle suspension component.
- The bending of a cantilever beam approximated near its tip.

Because only one coordinate governs the system's motion, the equations of motion reduce to a single second order differential equation. This simplicity allows us to derive closed form solutions for both free and forced responses, making SDOF systems a perfect teaching tool and a useful approximation for more complex structures.

## Key Parameters

Every SDOF system is characterized by three fundamental parameters:

1. Mass ( $m$ ) – the inertial property of the moving part.
2. Stiffness ( $k$ ) – the resistance to deformation, typically modeled by a linear spring.
3. Damping ( $c$ ) – the energy dissipation mechanism, often represented by a viscous damper.

From these, we derive two dimensionless quantities that describe the system's dynamic behavior:

- Natural frequency ( $\omega_n = \sqrt{k/m}$ )
- Damping ratio ( $\zeta = c/(2m\omega_n)$ )

These parameters determine how the system responds to external excitations, how quickly it settles, and whether it will exhibit resonance.

## Chapter 3 in Springer Textbooks: A Structured Overview

In the Springer edition of “Mechanical Vibrations,” **Chapter 3** systematically builds the foundation for SDOF analysis. It begins with the derivation of the governing equation, proceeds to solution methods for both undamped

and damped systems, and then explores forced vibrations and resonance phenomena. The chapter's structure mirrors the logical progression one would follow in a classroom:

1. Derivation of the mass–spring–damper equation.
2. Solution of the homogeneous (free) problem.
3. Analysis of damping regimes (underdamped, critically damped, overdamped).
4. Introduction of external forcing functions.
5. Frequency response and transfer function analysis.
6. Practical examples and problem sets.

By following this sequence, readers gain both the intuition and the analytical tools needed to tackle real world vibration problems.

## Mathematical Modeling of an SDOF System

### Derivation of the Equation of Motion

Consider a mass  $m$  attached to a linear spring of stiffness  $k$  and a linear damper of coefficient  $c$ . The external force acting on the mass is  $F(t)$ . Applying Newton's second law in the displacement direction  $x(t)$  yields:

$$m \ddot{x}(t) + c \dot{x}(t) + k x(t) = F(t)$$

Here,  $\ddot{x}(t)$  denotes the second derivative of displacement with respect to time (acceleration), and  $\dot{x}(t)$  denotes the first derivative (velocity). This linear, second order ordinary differential equation (ODE) encapsulates the entire dynamics of the SDOF system.

### Non Dimensionalization and Natural Frequency

It is often convenient to rewrite the equation in terms of natural frequency  $\omega_n = \sqrt{k/m}$  and damping ratio  $\zeta = c/(2m\omega_n)$ :

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \frac{F(t)}{m}$$

With  $\omega_n = \sqrt{k/m}$  (rad/s), we can directly interpret the system's behavior from these two parameters.

## Free (Homogeneous) Response

### Undamped Case ( $\zeta = 0$ )

When  $c = 0$ , the system oscillates indefinitely with constant amplitude:

$$x(t) = A \cos(\omega_n t) + B \sin(\omega_n t)$$

Using initial conditions  $x(0) = x_0$  and  $\dot{x}(0) = v_0$ :

- $A = x_0$
- $B = v_0 / \omega_n$

### Under Damped ( $0 < \zeta < 1$ )

For a lightly damped system, the solution decays exponentially while oscillating:

$$x(t) = e^{-\zeta\omega_n t} [ C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t) ]$$

where the damped natural frequency  $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ . The constants  $C_1$  and  $C_2$  are found from initial conditions.

### Critically Damped ( $\zeta = 1$ )

At critical damping, the system returns to equilibrium as quickly as possible without oscillating:

$$x(t) = (C_1 + C_2 t) e^{-\zeta \omega_n t}$$

### Over Damped ( $\zeta > 1$ )

When damping is heavy, the system slowly approaches equilibrium without oscillation:

$$x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

with  $r_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$ .

## Forced (Non Homogeneous) Response

### Steady State Solution for Harmonic Excitation

When the external force is harmonic,  $F(t) = F_0 \sin(\omega t)$ , the steady state response is also harmonic but with a phase shift:

$$x_{ss}(t) = X_0 \sin(\omega t - \phi)$$

where the amplitude  $X_0$  and phase  $\phi$  are given by:

- $X_0 = (F_0/m) / \sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}$
- $\tan \phi = (2\zeta\omega_n\omega) / (\omega_n^2 - \omega^2)$

These expressions reveal the classic resonance peak when  $\omega \rightarrow \omega_n$ , especially in lightly damped systems.

### Transfer Function and Frequency Response

Using Laplace transforms, the transfer function  $G(s)$  from force to displacement is:

$$G(s) = 1 / (m s^2 + c s + k)$$

Substituting  $s = i\omega$  gives the frequency response function:

$$G(i\omega) = \frac{1}{-m\omega^2 + i c \omega + k}$$

Plotting  $|G(i\omega)|$  versus  $\omega$  shows the resonance peak, while the angle of  $G(i\omega)$  gives the phase lag.

## Damping Mechanisms and Their Effects

Damping is the mechanism that dissipates vibrational energy. Common types include:

- Viscous damping – proportional to velocity ( $c \dot{x}$ )
- energy loss due to material hysteresis.
- non linear, often modeled with Coulomb or Stribeck laws.

Each mechanism alters the effective damping ratio  $\zeta$  and thus the system's response. In engineering practice, empirical damping ratios are often used because exact modeling can be complex.

## Practical Applications of SDOF Models

### Vehicle Suspension Design

Engineers approximate the vertical dynamics of a wheel assembly as an SDOF system. By tuning  $m$ ,  $k$ , and  $c$ , they can control ride comfort and handling performance.

## Seismic Isolation

Buildings and bridges are often modeled as SDOF systems for seismic analysis. The mass represents the structure's effective mass, the spring the stiffness of the isolation system, and the damper the energy dissipating device.

## Precision Instrumentation

Optical tables and scanning probe microscopes rely on SDOF models to design isolation mounts that suppress vibrations below a target frequency band.

## Mechanical Resonators

Micro electromechanical systems (MEMS) resonators are inherently SDOF, with natural frequencies in the MHz range. Understanding their damping and response is crucial for sensor accuracy.

## Numerical Example: A Vehicle Wheel Assembly

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