

chapter 4 exercise solutions principles of econometrics 3e

Published: September 27, 2026 | Index ID: #-

Unlocking Chapter 4: Exercise Solutions for Principles of Econometrics 3rd Edition

Welcome to the definitive guide to mastering Chapter 4 of the *Principles of Econometrics*, 3rd edition. Whether you're a graduate student tackling your first econometric assignment or a seasoned researcher revisiting foundational concepts, this article offers a clear, step by step walkthrough of the most common exercise problems. By blending concise explanations with thorough solutions, we'll help you internalize the principles that underpin the linear regression framework.

Why Chapter 4 Matters

Chapter 4 lays the groundwork for the entire text: it introduces the simple and multiple linear regression models, the Gauss Markov theorem, and the basics of hypothesis testing. A solid grasp of these topics is essential for the later chapters that cover model diagnostics, instrumental variables, panel data, and more advanced techniques.

What You'll Find Here

- Clear, human readable explanations of core concepts.
- Step by step solutions to the most frequently asked exercises.
- Practical tips for applying the theory to real data.
- Hints for extending the solutions to related problems.

Let's dive in.

Key Concepts Recap

Before tackling the exercises, it's useful to revisit the main ideas covered in Chapter 4:

1. The Linear Regression Model

The model is expressed as $y = X\beta + u$, where y is the dependent variable, X is a matrix of explanatory variables, β is a vector of coefficients, and u is the error term.

2. Ordinary Least Squares (OLS) Estimator

OLS finds the coefficient vector $\hat{\beta}$ that minimizes the sum of squared residuals: $\hat{\beta} = (X'X)^{-1}X'y$.

3. Gauss Markov Theorem

Under the classical assumptions (linearity, full rank, zero mean errors, homoskedasticity, no autocorrelation), the OLS estimator is the best linear unbiased estimator (BLUE).

4. Hypothesis Testing

We test linear restrictions on β using t -tests for single coefficients and F -tests for multiple restrictions.

5. Confidence Intervals

Standard errors of $\hat{\beta}$ are used to construct confidence intervals: $\hat{\beta} \pm t^*se(\hat{\beta})$.

Exercise Solutions

Exercise 4.1: Deriving the OLS Estimator

Problem: Show that the OLS estimator $\hat{\beta} = (X'X)^{-1}X'y$ solves the normal equations.

Solution:

1. Define the residual vector: $e = y - X\hat{\beta}$.
2. OLS minimizes the objective function: $Q(\hat{\beta}) = \sum_{i=1}^n (y_i - \hat{\beta}'x_i)^2 = (y - X\hat{\beta})'(y - X\hat{\beta})$.
3. Take the derivative with respect to $\hat{\beta}$: $\frac{\partial Q(\hat{\beta})}{\partial \hat{\beta}} = -2X'(y - X\hat{\beta}) = 0$.
4. Solve for $\hat{\beta}$: $X'X\hat{\beta} = X'y$, provided $X'X$ is invertible.

Thus, the OLS estimator satisfies the normal equations.

Exercise 4.2: Variance of the OLS Estimator

Problem: Derive the variance-covariance matrix of $\hat{\beta}$ under homoskedasticity.

Solution:

1. Recall that $\hat{\beta} = (\hat{\beta}_1, \dots, \hat{\beta}_k)'$.
2. Assuming $E(u) = 0$ and $\text{Var}(u) = \sigma^2 I_n$:
3. $\text{Var}(\hat{\beta}) = \text{Var}((X'X)^{-1}X'u) = (X'X)^{-1}X' \text{Var}(u)X(X'X)^{-1} = \sigma^2 (X'X)^{-1}$.
4. Thus, the variance-covariance matrix is $\sigma^2 (X'X)^{-1}$.

In practice, $\hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^n e_i^2$ is the residual mean square error.

Exercise 4.3: t Test for a Single Coefficient

Problem: Test the null hypothesis $H_0: \beta_j = 0$ against $H_a: \beta_j \neq 0$ using the t statistic.

Solution:

1. Estimate the regression to obtain $\hat{\beta}_j$ and its standard error $\text{se}(\hat{\beta}_j)$.
2. Compute the t statistic: $t = \hat{\beta}_j / \text{se}(\hat{\beta}_j)$.
3. Compare $|t|$ to the critical value from the t distribution with $n-2$ degrees of freedom.
4. If $|t|$ exceeds the critical value, reject H_0 at the chosen significance level.

Remember that the t test is symmetric; a negative $\hat{\beta}_j$ with a large magnitude still leads to rejection.

Exercise 4.4: F Test for Joint Significance

Problem: In a regression with three explanatory variables, test whether all coefficients except the intercept are jointly zero.

Solution:

1. Compute the restricted model (only intercept) and the unrestricted model (full regression).
2. Calculate the residual sum of squares (RSS) for both models: RSS_r and RSS_u .
3. Compute the F statistic: $F = \frac{(\text{RSS}_r - \text{RSS}_u) / q}{[\text{RSS}_u / (n - k)]}$, where $q = 3$ (number of restrictions) and $k = 4$ (total parameters including intercept).
4. Compare F to the critical value from the F distribution with q and $n - k$ degrees of freedom.
5. Reject the null if F exceeds the critical value.

This test tells you whether the explanatory variables collectively contribute to explaining variation in the dependent variable.

Exercise 4.5: Confidence Interval for a Linear Combination

Problem: Construct a 95% confidence interval for the linear combination $\beta_1 + 2\beta_2$.

Solution:

1. Estimate the regression to obtain $\hat{\beta}$, $\hat{\sigma}^2$, and the variance covariance matrix $\text{Var}(\hat{\beta})$.
2. Compute the point estimate: $\hat{y}_0 = \hat{\beta}'x_0$
3. Compute the standard error: $\text{se}(\hat{y}_0) = \sqrt{x_0' \text{Var}(\hat{\beta}) x_0}$.
4. Find the critical value from the t distribution: $t^* = t_{\alpha/2, n-2}$.
5. Construct the interval: $\hat{y}_0 \pm t^* \text{se}(\hat{y}_0)$.

Note that the covariance between $\hat{\beta}_0$ and $\hat{\beta}_1$ is essential for an accurate interval.

Exercise 4.6: Predicting with Confidence and Prediction Intervals

Problem: For a given value of $x = 5$, compute the predicted value and the associated 95% confidence and prediction intervals.

Solution:

1. Compute the predicted value: $\hat{y}_0 = \hat{\beta}'x_0$
 2. Compute the standard error of the prediction:
 3. For the confidence interval: $\text{se}_{\text{conf}} = \sqrt{\frac{1}{n} + (x_0 - \bar{x})' \frac{1}{\sum (x_i - \bar{x})^2} (x_0 - \bar{x})}$
 4. For the prediction interval: $\text{se}_{\text{pred}} = \sqrt{1 + \frac{1}{n} + (x_0 - \bar{x})' \frac{1}{\sum (x_i - \bar{x})^2} (x_0 - \bar{x})}$
- Confidence interval: $\hat{y}_0 \pm t^* \text{se}_{\text{conf}}$
 - Prediction interval: $\hat{y}_0 \pm t^* \text{se}_{\text{pred}}$

Exercise 4.7: Testing for Heteroskedasticity (Breusch Pagan)

Problem: Conduct the Breusch Pagan test on the residuals from a regression.

Solution:

1. Obtain the OLS residuals: e_i
2. Compute the auxiliary regression: $e_i = \alpha + \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i$
3. Calculate the explained sum of squares (ESS) from this regression.
4. Compute the test statistic: $LM = n \cdot R^2_{\text{aux}}$
5. Under the null of homoskedasticity, LM follows a chi square distribution with k degrees of freedom.
6. Reject the null if LM exceeds the critical chi square value.

A significant result indicates that the variance of the errors depends on the explanatory variables.

Exercise 4.8: White's Test for Heteroskedasticity

Problem: Perform White's test using squared residuals and cross product terms.

Solution:

1. Generate the set of regressors for the auxiliary regression: include all original regressors, their squares, and cross products.
2. Run the auxiliary regression of e_i^2 on the expanded set.

Baca Juga (Artikel Terkait)

- [chapter 4 atomic structure wordwise answer key](#)

URL: <http://www.bbgglobalsolutions.com/chapter-4-atomic-structure-wordwise-answer-key.pdf>

- [chapter 4 atomic structure test a answers](#)

URL: <http://www.bbgglobalsolutions.com/chapter-4-atomic-structure-test-a-answers.pdf>

- [chapter 4 atomic structure section 4 1 studying atoms](#)

URL: <http://www.bbgglobalsolutions.com/chapter-4-atomic-structure-section-4-1-studying-atoms.pdf>

- [chapter 4 atomic structure henry county school](#)

URL: <http://www.bbgglobalsolutions.com/chapter-4-atomic-structure-henry-county-school.pdf>

Tags: #chapter #exercise #solutions #principles #econometrics

