

characteristics of quadratic functions answer key

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Introduction

Quadratic functions are a cornerstone of algebra and calculus, appearing in everything from projectile motion to financial forecasting. Whether you're a student tackling a worksheet, a teacher designing a lesson plan, or an enthusiast exploring mathematical patterns, a clear grasp of the characteristics of quadratic functions is essential. This article provides a comprehensive guide to those characteristics, explains how to identify them quickly, and offers a detailed answer key for common practice problems. By the end, you'll have both the conceptual framework and the practical tools needed to master any quadratic function challenge.

What Is a Quadratic Function?

A **quadratic function** is any function that can be expressed in the form

$$f(x) = ax^2 + bx + c$$

where **a**, **b**, and **c** are constants and **a** ≠ 0. The graph of a quadratic function is a parabola, a U-shaped curve that opens upward if **a** is positive and downward if **a** is negative.

Key Features of the Standard Form

- **a** – Determines the direction and width of the parabola.
- **b** – Influences the horizontal placement of the vertex and the axis of symmetry.
- **c** – Represents the y-intercept, the point where the parabola crosses the y-axis.

Vertex Form and Its Advantages

While the standard form is useful for algebraic manipulation, the vertex form makes it easier to read the parabola's key properties. The vertex form is

$$f(x) = a(x - h)^2 + k$$

where **(h, k)** is the vertex of the parabola. Converting between the two forms involves completing the square, a technique that also reveals the discriminant.

Finding the Vertex

Given the standard form, the vertex coordinates are calculated as:

$$h = -b / (2a) \text{ and } k = f(h)$$

Once **h** is known, substitute it back into the function to find **k**. This method is quick and reliable, especially when solving for the vertex in a test setting.

Axis of Symmetry

The **axis of symmetry** is the vertical line that divides the parabola into two mirror images. Its equation is simply

$$x = -b / (2a)$$

In the vertex form, the axis of symmetry is $x = h$, making it immediately apparent from the equation.

Y Intercept and X Intercepts

The y intercept is straightforward: set $x = 0$ in the function:

$$f(0) = c$$

For the x intercepts (roots), solve the quadratic equation $ax^2 + bx + c = 0$. The solutions depend on the discriminant, which is explained in the next section.

Discriminant: Decoding the Roots

The **discriminant** is the part of the quadratic formula under the square root:

$$b^2 - 4ac$$

It tells you the nature of the roots:

- $b^2 - 4ac > 0$ Two distinct real roots.
- $b^2 - 4ac = 0$ One real root (the parabola touches the x axis at the vertex).
- $b^2 - 4ac < 0$ Two complex conjugate roots (the parabola never crosses the x axis).

Using the Quadratic Formula

When the discriminant is known, the roots are found via

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In a timed test, it's often quicker to factor the quadratic when possible, but the formula guarantees a solution even for non-factorable equations.

Graphing a Quadratic Function

Plotting a quadratic function involves several steps that reinforce understanding of its characteristics:

1. Plot the y intercept (0, c).
2. Find the vertex (h, k) and plot it.
3. Locate the axis of symmetry $x = h$.
4. Determine the x intercepts using the discriminant or factorization.
5. Plot additional points for accuracy.
6. Sketch the parabola, ensuring it opens in the correct direction based on the sign of a.

By systematically marking these key points, the graph becomes a visual representation of the function's underlying algebraic properties.

Common Mistakes and How to Avoid Them

- Ignoring the sign of a – Misidentifying the opening direction leads to incorrect graphing.
- Incorrectly computing the vertex – Forgetting the factor of 2 in the denominator for h.
- Assuming all quadratics cross the x axis – Overlooking the discriminant's role in determining real roots.
- Failing to check for symmetry – Skipping the axis of symmetry can distort the plotted curve.

Practical Applications of Quadratic Functions

Quadratic functions model real world phenomena where acceleration or curvature is involved. Some classic examples include:

- Projectile Motion – The height of an object thrown upward follows a quadratic trajectory.
- Economics – Profit and cost functions often have quadratic terms reflecting economies of scale.
- Physics – The relationship between potential energy and displacement for a spring is quadratic.
- Engineering – Structural designs, such as arches, rely on parabolic curves for optimal load distribution.

Sample Problems and Answer Key

Below are five practice problems that cover a range of quadratic function characteristics. The answer key follows each question, providing step by step solutions.

Problem 1: Identify the Vertex and Axis of Symmetry

Given $f(x) = 2x^2 - 8x + 6$, find the vertex and the axis of symmetry.

Answer Key – Problem 1

- Compute $h = -b/(2a) = -(-8)/(2 \cdot 2) = 8/4 = 2$.
- Find $k = f(2) = 2(2)^2 - 8(2) + 6 = 8 - 16 + 6 = -2$.
- Vertex: $(2, -2)$.
- Axis of symmetry: $x = 2$.

Problem 2: Determine the Discriminant and Roots

For $g(x) = 3x^2 + 6x + 3$, calculate the discriminant and the real roots.

Answer Key – Problem 2

- Discriminant: $9b^2 - 4ac = 9(6)^2 - 4(3)(3) = 324 - 36 = 288$.
- Since $9b^2 - 4ac > 0$, there are two distinct real roots.
- $x = -b/(2a) = -6/(2 \cdot 3) = -1$.
- Root: $x = -1$ (double root).

Problem 3: Convert to Vertex Form

Rewrite $h(x) = x^2 - 4x + 5$ in vertex form.

Answer Key – Problem 3

- Complete the square: $x^2 - 4x + 5 = (x^2 - 4x + 4) + 1 = (x - 2)^2 + 1$.
- Vertex form: $h(x) = (x - 2)^2 + 1$.
- Vertex: $(2, 1)$.

Problem 4: Find the Y Intercept and X Intercepts

Given $p(x) = -x^2 + 6x - 8$, determine the y intercept and all x intercepts.

Answer Key – Problem 4

- Y intercept: $p(0) = -0 + 0 - 8 = -8$!' point $(0, -8)$.
- Discriminant: $9b^2 - 4ac = 9(6)^2 - 4(-1)(-8) = 324 - 32 = 292$.
- Roots: $x = [-6 \pm \sqrt{292}]/(2 \cdot -1) = [-6 \pm 2\sqrt{73}]/-2$.
- Compute two roots:
- $x_1 = (-6 + 2\sqrt{73})/-2 = (-4 + \sqrt{73})/-2 = 2$.
- $x_2 = (-6 - 2\sqrt{73})/-2 = (-8 - \sqrt{73})/-2 = 4$.
- X intercepts: $(2, 0)$ and $(4, 0)$.

Problem 5: Sketch the Parabola Using Key Points

Using the function $q(x) = 0.5x^2 - 3x + 4$, identify all key points and sketch the graph.

Answer Key – Problem 5

- Y intercept: $q(0) = 4$!' (0, 4).
- Vertex: $h = -b/(2a) = -(-3)/(2 \cdot 0.5) = 3/1 = 3$.
- Compute $k = q(3) = 0.5(3)^2 - 3(3) + 4 = 4.5 - 9 + 4 = -0.5$.
- Vertex: (3, -0.5).
- Axis of symmetry: $x = 3$.
- Discriminant: $9B \hat{O} , \acute{O}2>" \hat{O} Bf \tilde{a}R'fB' \hat{O} ' \hat{O} , \hat{O} \grave{a}$
- Roots: $x = [3 \pm " 1]/(2 \cdot 0.5) = [3 \pm 1]/1$.
- Compute two roots:
- $x \bullet = (3 + 1)/1 = 4$!' (4, 0).
- $x , = (3 - 1)/1 = 2$!' (2, 0).

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